Electrical and Electronics Engineering
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Master Semester 2

Course
Smart grids technologies

The per-unit method

Prof. Mario Paolone
Distributed Electrical Systems Laboratory
École Polytechnique Fédérale de Lausanne (Switzerland)

Introduction

A **per-unit system** is obtained by referring all quantities of a system to **base values**.

Let $\bar{A} = A_{re} + jA_{im}$ be a complex quantity, and A_b the its base value. The corresponding per-unit quantity $\bar{a}$ is obtained as

$$\bar{a} = \frac{\bar{A}}{A_b} = \frac{A_{re}}{A_b} + j\frac{A_{im}}{A_b} = a_{re} + ja_{im}$$

For the abslute value $|\bar{a}|$, one finds

$$|\bar{a}| = \sqrt{a_{re}^2 + a_{im}^2} = \sqrt{\left(\frac{A_{re}}{A_b}\right)^2 + \left(\frac{A_{im}}{A_b}\right)^2} = \frac{\sqrt{A_{re}^2 + A_{im}^2}}{A_b} = \frac{|\bar{A}|}{A_b}$$

Coherent Base

When selecting the base values for a number of quantities that are coupled by physical laws, the dependencies between them need to be respected. That is, the chosen base values need to be **coherent** (i.e., respect the same physical laws).

Example: Let A, B, C be quantities which are related by

$$C = \frac{A}{B}$$

Then, the corresponding base values A_b, B_b, C_b need to satisfy the same relation in order to form a coherent base. Namely

$$C_b = \frac{A_b}{B_b}$$

Single-Phase Case

In power system analysis, one needs to select base values for **power**, **voltage**, **current**, and **impedance** (or **admittance**).

In a single-phase system, the base power A_b and the base voltage V_b are (obviously) single-phase quantities. Typically, A_b is the rated **per-phase power**, and V_b the **phase-to-ground voltage**. The other base values can be computed as

$$I_b = \frac{A_b}{V_b}$$

$$Z_b = \frac{V_b}{I_b} = \frac{V_b^2}{A_b} \left(Y_b = \frac{1}{Z_b} = \frac{A_b}{V_b^2} \right)$$

Three-Phase Case

In three-phase systems, it is more natural to select A_b as the **three-phase power**, and V_b as the **phase-to-phase voltage**. The other base values are computed as

$$I_b = \frac{A_b}{\sqrt{3}V_b}$$

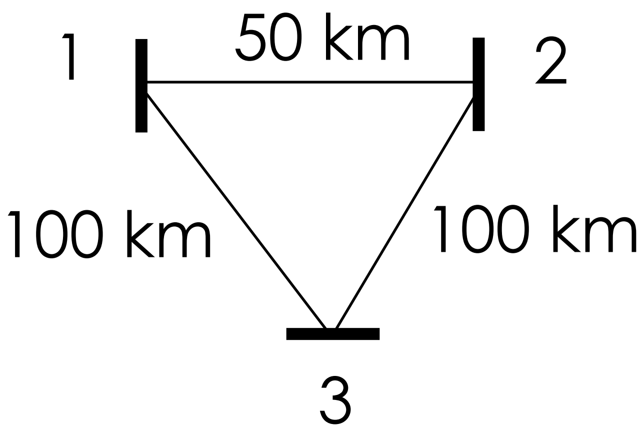
$$Z_b = \frac{E_b}{I_b} = \frac{V_b}{\sqrt{3}I_b} = \frac{V_b^2}{A_b} \left(Y_b = \frac{1}{Z_b} = \frac{A_b}{V_b^2} \right)$$

If the system is balanced, it can be reduced to an equivalent single-phase system (i.e., the positive-sequence system).

The phase-to-phase base voltage V_b and the phase-to-ground base voltage E_b are related as follows

$$E_b = \frac{V_b}{\sqrt{3}}$$

Example



Line parameters:

$R' = 0{,}0717\ \Omega/km$

$X' = 0{,}424\ \Omega/km$

$G' = 0\ S/km$

$B' = 2{,}64\ \mu S/km$

Line rated voltage: $220\ kV$.

Assume a base power of 100 MW.

Calculate the nodal admittance matrix in pu.

Example

The base values are $P_b = 100\ MW$ and $V_b = 220\ kV$.

First, transform the longitudinal and transversal electrical parameters of the lines into pu. Namely

$$r' = R' \frac{A_b}{V_b^2} = 0.0717 \cdot \frac{100}{220^2} \approx 1.467 \cdot 10^{-4}\ \text{pu/km}$$

$$x' = X' \frac{A_b}{V_b^2} = 0.424 \cdot \frac{100}{220^2} \approx 8.760 \cdot 10^{-4}\ \text{pu/km}$$

$$b' = B' \frac{V_b^2}{A_b} = 2.64\mu \cdot \frac{220^2}{100} \approx 1.278 \cdot 10^{-3}\ \text{pu/km}$$

Example

Then, compute the branch admittances in p.u.

$$\bar{y}_{12} = \frac{1}{50(r' + jx')} \approx 3.75 - j22.2$$

$$\bar{y}_{13} = \frac{1}{100(r' + jx')} \approx 1.88 - j11.1$$

$$\bar{y}_{23} = \frac{1}{100(r' + jx')} \approx 1.88 - j11.1$$

and shunt admittances, too

$$\bar{y}_{10} = \frac{1}{2}(50 + 100)jb' \approx j0.095$$

$$\bar{y}_{20} = \frac{1}{2}(50 + 100)jb' \approx j0.095$$

$$\bar{y}_{30} = \frac{1}{2}(100 + 100)jb' \approx j0.128$$

Example

The entries of the per-unit nodal admittance matrix are

$$\bar{Y}_{11} = \bar{y}_{12} + \bar{y}_{13} + \bar{y}_{10} \approx 5.63 - j33.3$$

$$\bar{Y}_{22} = \bar{y}_{12} + \bar{y}_{23} + \bar{y}_{20} \approx 5.63 - j33.3$$

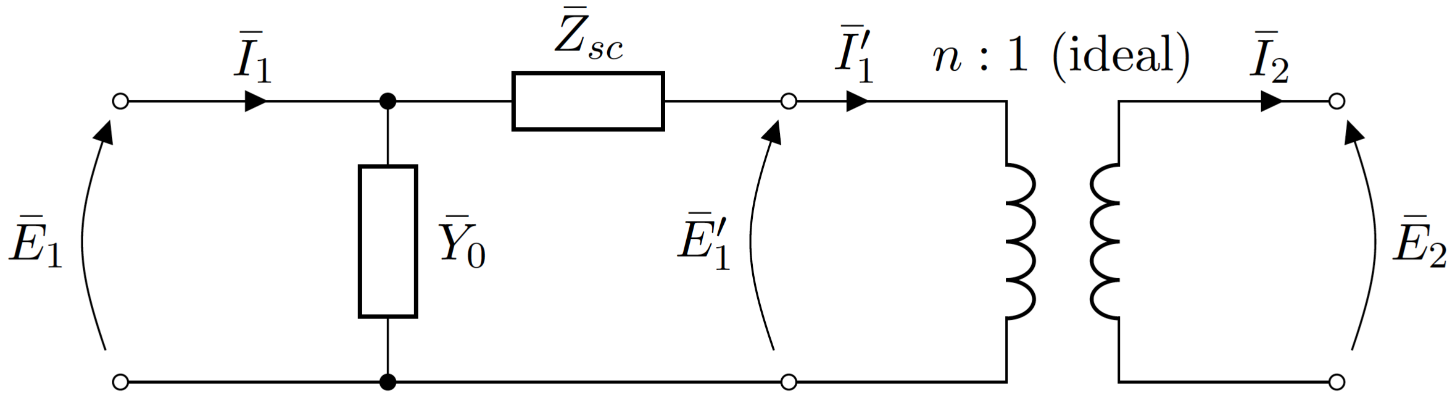
$$\bar{Y}_{33} = \bar{y}_{13} + \bar{y}_{23} + \bar{y}_{30} \approx 3.75 - j22.1$$

$$\bar{Y}_{12} = \bar{Y}_{21} = -\bar{y}_{12} \approx -3.75 + j22.2$$

$$\bar{Y}_{13} = \bar{Y}_{31} = -\bar{y}_{13} \approx -1.88 + j11.1$$

$$\bar{Y}_{23} = \bar{Y}_{32} = -\bar{y}_{23} \approx -1.88 + j11.1$$

Equivalent Circuit of a Transformer



The equivalent circuit of a transformer in absolute units looks as shown above, where $\bar{Z}_{sc}$ is the **short-circuit impedance**, $\bar{Y}_0$ is the **zero-load admittance**, and n is the **transformation ratio**.

The transformer is described by

$$\bar{E}_1 = n\bar{E}_2 + \frac{\bar{Z}_{sc}}{n}\bar{I}_2$$

$$\bar{I}_1 = n\bar{Y}_0\bar{E}_2 + \frac{1}{n}(1 + \bar{Y}_0\bar{Z}_{sc})\bar{I}_2$$

Equivalent Circuit of a Transformer

Let's now consider the base values V_{b1}, V_{b2} equal to the line-to-line voltage for the two sides of the transformer and the base value A_b equal to the three-phase power. The parameters of the transformer Y_0 and Z_{sc}, expressed in per-unit, will be as follow:

$$z_{sc} = Z_{sc}\frac{A_b}{V_{b_1}^2} \qquad\qquad y_0 = Y_0\frac{V_{b_1}^2}{A_b}$$

It is now possible to write the previous equations in per-unit by multiplying it by $\frac{\sqrt{3}}{V_{b1}}$. Let's start with the first one:

$$\bar{E}_1 \cdot \frac{\sqrt{3}}{V_{b_1}} = n\bar{E}_2 \cdot \frac{\sqrt{3}}{V_{b_1}} \cdot \frac{V_{b_2}}{V_{b_2}} + \bar{Z}_{sc} \cdot \frac{\bar{I}_2}{n} \cdot \frac{\sqrt{3}}{V_{b_1}}$$

$$\bar{v}_1 = n\bar{v}_2\frac{V_{b2}}{V_{b1}} + \bar{z}_{sc}Z_{b1} \cdot \frac{\bar{\iota}_2 I_{b2}}{n}\frac{\sqrt{3}}{V_{b1}} = n\bar{v}_2\frac{V_{b2}}{V_{b1}} + \bar{z}_{sc}\frac{V_{b1}^2}{A_b} \cdot \frac{\bar{\iota}_2}{n}\frac{A_b}{\sqrt{3}V_{b2}}\frac{\sqrt{3}}{V_{b1}}$$

$$\bar{v}_1 = n\bar{v}_2\frac{V_{b_2}}{V_{b_1}} + \bar{z}_{sc}\frac{\bar{\iota}_2}{n}\frac{V_{b_1}}{V_{b_2}}$$

Equivalent Circuit of a Transformer

Similarly, the second equation of the transformer can be re-written in per-unit as follow:

$$\bar{I}_1 \cdot \frac{\sqrt{3}V_{b_1}}{A_b} = \bar{Y}_0(n\bar{V}_2) \cdot \frac{\sqrt{3}V_{b_1}}{A_b} \cdot \frac{V_{b_1}}{V_{b_1}} \cdot \frac{V_{b_2}}{V_{b_2}} + (\bar{Y}_0\bar{Z}_{sc} + 1) \cdot \frac{\bar{I}_2}{n} \cdot \frac{\sqrt{3}V_{b_1}}{A_b}$$

$$\bar{\imath}_1 = \bar{y}_0(n\bar{v}_2) \cdot \frac{V_{b_2}}{V_{b_1}} + \left(\bar{y}_0 \frac{A_b}{V_{b1}^2} \bar{z}_{sc} \frac{V_{b1}^2}{A_b} + 1\right) \frac{\bar{\imath}_2}{n} \frac{A_b}{\sqrt{3}V_{b2}} \frac{\sqrt{3}V_{b_1}}{A_b}$$

$$\bar{\imath}_1 = \bar{y}_0(n\bar{v}_2) \cdot \frac{V_{b_2}}{V_{b_1}} + (\bar{y}_0\bar{z}_{sc} + 1) \cdot \frac{\bar{\imath}_2}{n} \cdot \frac{V_{b_1}}{V_{b_2}}$$

Equivalent Circuit of a Transformer

If the base voltages V_{b1} and V_{b2} on the **primary** and **secondary** side of the transformer are chosen such that

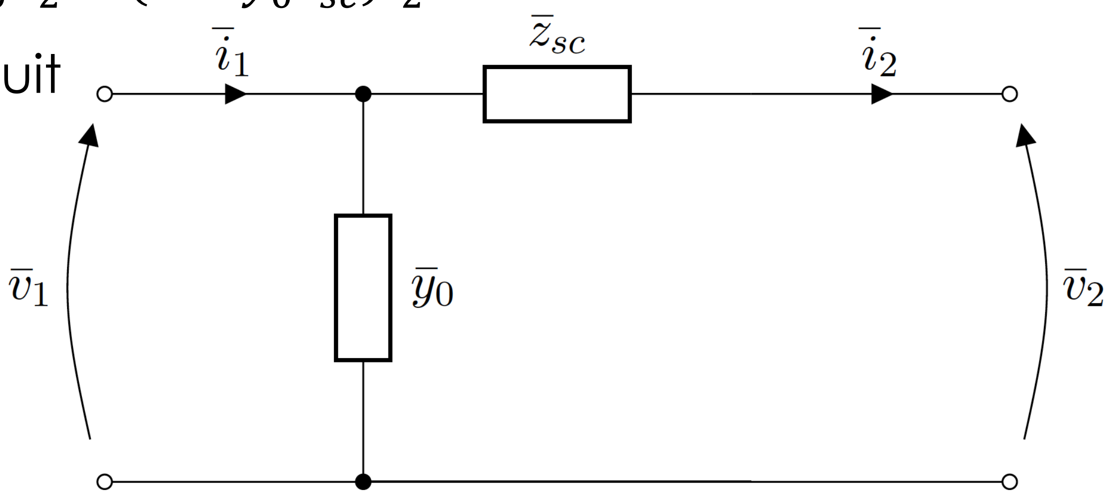
$$\frac{V_{b1}}{V_{b2}} = n$$

(i.e., their ratio matches the transformation ratio), the per-unit transformer equations read as follows

$$\bar{v}_1 = \bar{v}_2 + \bar{z}_{sc}\bar{\imath}_2$$

$$\bar{\imath}_1 = \bar{y}_0\bar{v}_2 + (1 + \bar{y}_0\bar{z}_{sc})\bar{\imath}_2$$

This corresponds to the circuit



Equivalent Circuit of a Transformer

Conversely, if V_{b1} and V_{b2} are selected so that

$$\frac{V_{b1}}{V_{b2}} \neq n$$

the equivalent circuit is more complicated. Let z'_{sc} and y'_0 be the values of Z_{sc} and Y_0 expressed in the base of the primary side, and $m = n\left(\frac{V_{b1}}{V_{b2}}\right)^{-1}$. Then, the equivalent circuit looks as follows